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101. Substitute $(-2, 17)$ to form an equation.

102. Let $x = 0.\dot{4}\dot{2}$. Write down $x = 0.424242...$ and then express $100x$ similarly. Subtract to find $99x$ and make x the subject.

103. The four probabilities must add to 1. Write this as an equation in a, and solve.

104. The circle is centred at the origin, and the square of its radius is 10. Using Pythagoras, find the squared distance between the origin and $(2, 3)$. Compare this to 10.

105. (a) The law starts $\left(a^b\right)^c \equiv ...$
 (b) Simplify each factor using the definition of a logarithm, which is that $\log_a b$ is "the power you need to raise a by to get b".

106. A unit vector is defined as a vector with unit length, i.e. a vector with length/magnitude 1. Use 3D Pythagoras to find the length.

107. (a) The object is assumed to have negligible size, i.e. it is assumed to be a *insert technical term.* Gravity is taken to be the only force acting, so $a =$ *insert magnitude and direction.*
 (b) When it lands, the vertical displacement of the projectile from its initial launch point is 0. So, substitute $s = 0$, $u = 19.6$ and $a = -9.8$ and solve for the time of flight t.
 (c) Take the time of flight from the previous part, and using it in a horizontal

$$\text{distance} = \text{speed} \times \text{time}.$$

108. Use the polynomial differentiation formula

$$y = x^n \implies \tfrac{dy}{dx} = nx^{n-1}.$$

Having found an expression for $\frac{dy}{dx}$, substitute the value $x = 1$ to find the gradient of the tangent. Then use the formula for the equation of a straight line

$$y - y_1 = m(x - x_1).$$

109. Express 8 as a power of 2. Then reverse the order of the indices using the index law

$$\left(a^b\right)^c \equiv a^{bc}.$$

110. The percentage error in an estimation of a quantity s is given by

$$\frac{s_{\text{estimated}} - s_{\text{actual}}}{s_{\text{actual}}}.$$

Set the angle unit on your calculator to radians for evaluation of s_{actual}.

111. Use the factor theorem regarding a polynomial $\text{f}(x)$. In this case, the factor theorem is as follows:

$$(bx - a) \text{ is a factor of } \text{f}(x) \iff \text{f}\left(\tfrac{a}{b}\right) = 0.$$

So, write down the three roots of the equation (two of them in terms of a) and then compare to the roots $x = 3$ and $x = 6$.

112. Solve the equation $x^2 - 5x = 0$. Then sketch the parabola $y = x^2 - 5x$. Locate the set of x values (regions of the x axis) for which the parabola is below the x axis.

113. Use NII to set up a quadratic in k. The resultant force is $(k^2 - 1) - (k + 1)$ N.

114. The rule for integration, which can be applied term by term, is

$$\int kx^n \, dx = \frac{k}{n+1} x^{n+1} + c.$$

115. Any parabola of the form $y = ax^2 + bx + c$ has a line of symmetry parallel to the y axis. Draw a sketch, establishing that the points of intersection are reflections of each other in this line. With a sentence of explanation, this proves the result.

116. Find the prime factors of 28. Taking all possible combinations of these gives the set of divisors.

117. In radians, the sum of the interior angles of an n-gon is given by $\pi(n - 2)$. Evaluate this for $n = 5$, and set up and solve an equation.

118. In the direction of the implication, square roots or cube roots have been taken (and the equations have been turned around). For a counterexample, try $(-2, 4)$.

119. Three lines are "concurrent" if they all intersect at the same point. So, find the intersection of the first two, then, by substituting this point in, find k such that the third line passes through that point.

120. Write in the form $(3^x + ...)(3^x - ...) = 0$, and use the factor theorem. Alternatively, let $z = 3^x$ and solve a regular quadratic in z.

121. Solve simultaneously, by substituting for y. The resulting equation can be factorised.

122. As with any identity, you should start with one side, as an expression. Consider the LHS alone. Put it over a common denominator and simplify to reach the RHS.

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123. (a) Multiply out and simplify the algebra.
     (b) Consider the LHS and the RHS as describing the squared distance from (x, y) to the points $(0, 0)$ and $(1, 1)$ respectively.

124. If variables p and q are "related linearly", then $p = mq + c$, for some constants m, c. In the question, divide both sides by y to produce the relevant variables.

125. Set up equations in x and y, one for perimeter and one for area. Solve simultaneously.

126. Suppose one card has been dealt. It makes no difference what this card is, so call it the ace of spades. Consider the probability that the second card dealt is also an ace. Comparing the value of this probability in the two cases will give you the answer.

127. This is a difference of two squares $(p + q)(p - q)$, where $p = x - 2$ and $q = \sqrt{11}$.

128. (a) Put in the form $ax^2 + bx + c = 0$ and consider the quadratic discriminant.
     (b) Use part (a) in your sketch.
     (c) Output values are plotted as y values. And the function whose outputs are equal to its inputs is $x \mapsto x$. Hence, you need to show that the parabola is always above the line $y = x$.

129. Write the numbers as $2a+1$ and $2b+1$, for $a, b \in \mathbb{Z}$.

130. Substitute the point $(1, b)$ into both lines, and solve the resulting pair of simultaneous equations.

131. Differentiate both sides with respect to x.

132. Use the factor theorem: if $(x - \alpha)$ is a factor, then $x = \alpha$ is a root.

133. A fraction can only be zero when its numerator is zero. So, write down the roots of the numerator.

134. Consider $x = -1$, testing if it satisfies $x^2 = 1$ and/or $x^5 = 1$. It is a counterexample to one of the implications.

135. The boundary equation is $3x^2 + 6x + 1 = 0$. Solve this using the quadratic formula, finding the roots as decimals. Values of x satisfying the inequality lie between these roots.

136. Each tile can be placed independently in one of two orientations, so the possibility space consists of $2^4 = 16$ outcomes. Count up the number of successful outcomes in each case.

137. Differentiate to find the gradient formula $\frac{dy}{dx}$. Then evaluate $\frac{dy}{dx}$ at $x = 2$.

138. Establish that there are two ways of placing the heavy books. Then consider the number of ways of arranging the other 8 amongst themselves.

139. Consider the value $x = 2$. It is a counterexample to one direction of implication.

140. Use Pythagoras.

141. (a) The force diagram should have two forces and an acceleration. There is no need to include units on a force diagram (and it's a good idea not to, they just get in the way), as long as the standard conventions are used.
     (b) Use NII.
     (c) Use NIII.

142. Draw the longer diagonal in, subdividing the kite into two congruent triangles. Calculate the area using $\frac{1}{2}bh$.

143. Substitute values and solve the resulting pair of simultaneous equations. At the end, be careful to use the fact that the roots are distinct.

144. The number 15 springs to mind, but is wrong!

145. Multiply both sides by $(x - 1)^2$, then multiply out the brackets. Equate everything to zero, and solve by factorising.

146. (a) The notation means "$16x^2 - 44x + 6$ evaluated at $x = \frac{1}{2}$".
     (b) In the factor theorem, $(2x - 1)$ corresponds to the root $x = \frac{1}{2}$.

147. Rearrange the quadratic into the usual form, equalling zero. Then calculate the discriminant $\Delta = b^2 - 4ac$, showing that it is negative.

148. The language of this question may look heavy, but the calculations involved aren't. All you need to do is differentiate the given function. The easiest way to describe the derivative f' is with $f'(x) = ...$

149. Quote or else use your calculator to find $\arcsin \frac{\sqrt{3}}{2}$. Then consult a unit circle for the second angle θ.

150. Use the factor theorem, keeping in mind that a constant factor will also be necessary to ensure the parabola passes through $(0, -12)$

151. Angles in a triangle add up to π radians.

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152. (a) This is a positive parabola with x intercepts at 0 and a.
 (b) This is a negative parabola.
 (c) This is a positive cubic with a double root at $x = 0$ and a single root at $x = a$. So, the curve just touches the x axis at $x = 0$ and crosses it at $x = a$.

153. You can get rid of the modulus function by using the implication

$$|a| = b \implies a = \pm b.$$

To use this, rearrange to make $|10-3x|$ the subject of the equation.

154. Define the dimensions of the lawn as x and y, and set up simultaneous equations. Substitute for y, and convert the resulting equation into a quadratic in x^2.

155. An arithmetic progression (AP) is one in which there is a common (constant) difference between successive terms. Write b, c, d in terms of a and the common difference. The common difference is usually notated d, but that's used in the question, so call it k.

156. If you need to visualise this, you might sketch the sets on a number line. Note that, in interval set notation, square brackets signify inclusion of the boundary, while round brackets signify exclusion of the boundary.

157. Calculate squared distances using Pythagoras.

158. The number of possible orders of n distinct objects is $n!$. Only one of these is successful.

159. The language here is the same as asking for the vertex or SP of the parabola $y = x(x - 5)$.

160. Take out a common factor of x^2.

161. Find the y coordinates of the endpoints of the chords. Then find the gradient, and use

$$y - y_1 = m(x - x_1).$$

162. Solve $\text{f}(x) = 100$ and $\text{g}(x) = 100$.

163. (a) Take out square factors from the surds.
 (b) Put the fractions over a common denominator.

164. Use the factorial definition

$$^n\text{C}_r = \frac{n!}{r!(n-r)!}.$$

To simplify, use the fact that

$$\frac{n!}{(n-2)!} \equiv \frac{n(n-1)(n-2)...(2)(1)}{(n-2)...(2)(1)} \equiv n(n-1).$$

Take care to check that all values found do, in fact, give a well defined $^n\text{C}_2 = 15$.

165. The process of finding all possible functions with a given derivative is integration. Remember the constant of integration.

166. Get rid of the fractions first: multiply both sides of the equation by $x + 1$ and x. Then solve the resulting quadratic.

167. The boundary equation is a circle.

168. Calculate the distances of the endpoints from the origin. The point furthest from the origin must be one of these.

169. Consider the change in the resultant force when the smallest force is removed.

170. Multiply top and bottom by the conjugate of the bottom, which, in this case, is $5 + 2\sqrt{6}$.

171. (a) Substitute $4 = 2^2$ and use an index law to switch the order of the indices.
 (b) The equation is a quadratic in x.
 (c) Use the factor theorem, noting that one of the factors does not produce any roots.

172. The two sets are complements of one another, meaning that they partition the space exactly into two regions. Locating these two regions on a Venn diagram is enough to show that they are mutually exclusive.

— Alternative Method —

Assume that x is an element of both $A' \cup B$ and $A \cap B'$. Find an explicit contradiction to show that there is no element in both.

173. Write the area as a single integral.

— Alternative Method —

Sketch the scenario and use the formula for the area of a triangle.

174. (a) Show that $x = 2$ is a root.
 (b) Take out the factor of $(x - 2)$, then factorise the remaining quadratic.
 (c) Use the factor theorem.

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175. "Verify" means "check that it works". So, find the relevant pairs of primes for $n = 4, 6, ..., 20$.

176. The gradients m_1, m_2 of perpendicular lines are negative reciprocals. Equivalently, $m_1 m_2 = -1$.

177. A linear function is $\text{f}(x) = ax + b$. Use the value of $\text{f}'(1)$ to find a (the gradient of a straight line is constant, so the gradient at any point e.g. $x = 1$ gives the gradient everywhere). Then use the value of $\text{f}(1)$ to find b.

178. The odd integers form an AP. Work out the first term a and the common difference d, and then sub them into the formula.

179. Firstly, sketch the curve $x = y^2$. Then work out which points on that curve also appear on $y = \sqrt{x}$.

180. The possibility space is the same in both cases, consisting of $6^2 = 36$ equally likely outcomes. So, you need only compare the number of successful outcomes.

181. Set up and solve a pair of simultaneous equations for intersections. Then consider carefully whether any points of intersection you find are necessarily *crossing* points.

182. The odd and even cases are different.

183. (a) Complete the square for x and for y.
  (b) Use the fact that C_1 is symmetrical in $y = x$, and passes through the origin.
  (c) Consider the centre and radius of C_2, then turn this back into an equation.
  (d) By definition, both circles must be tangent to the line $y = -x$, one on either side of it.

184. Rationalise the denominator, by multiplying top and bottom by the conjugate of the bottom.

185. (a) Since the object is in equilibrium, the sum of the three forces (as vectors) is zero.
  (b) Note that $(45, 60, 75)$ is a Pythagorean triple.

186. (a) Using the equation of the parabola, find the x coordinates of the points of intersection. Then substitute both points into $y = mx + c$ to form two simultaneous equations in m and c.
  (b) Solve by elimination or substitution.

187. In both parts, multiply out the brackets first.

188. Consider the single root at $x = a$ and double root at $x = b$.

189. Write down the probability that the first name drawn out is the first name alphabetically. Then consider the second name and the third name. Multiply the three probabilities together.

190. Consider the sum of all five vectors.

191. If in doubt, plug a large number into a calculator and see what happens!

192. Set up two equations for horizontal equilibrium and vertical equilibrium, and solve simultaneously.

193. Name the dimensions x and y. Set up equations for perimeter and area and solve simultaneously for x and y. Then find the lengths of the diagonals by Pythagoras.

194. The relevant formulae are
  (a) $\mathbb{P}(A \cap B) = \mathbb{P}(A) \times \mathbb{P}(B)$.
  (b) $\mathbb{P}(A \cup B) \equiv \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$.

195. This can be done by calculus, but the question says no. Solve simultaneously for intersections, factorising the resulting equation. You could also use Δ to show that there is a double root.

196. In each case, the answer is one word. An accurate sketch should give you the shape.

197. Integrate to find y, remembering the constant of integration. Then substitute the point $(0, 3)$ to find the $+c$.

198. Multiply by $(1 - x)$. Then equate the coefficients of x and equate the constant terms.

199. Both are straight lines. The form in which they appear is
$$\frac{y - b}{x - a} = m.$$
This passes through (a, b) with gradient m.

200. Use scale factors. Write down the scale factor from the arc to the circumference, then scale the angle subtended by the arc by this.

——— End of 2nd Hundred ———

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